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PUBLIC DEBT AND REDISTRIBUTION WITH BORROWING CONSTRAINTS*

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We build a model with financial imperfections and heterogeneous agents and analyse the effects of two types of fiscal policy: revenue-neutral, intratemporal redistribution; and debt-financed tax cuts, which we interpret as intertemporal redistribution. Under flexible prices, the two policies are either neutral or display effects that are at odds with the empirical evidence. With sticky prices, Ricardian equivalence always fails. A Robin Hood, revenue-neutral redistribution to borrowers is expansionary on aggregate activity. A uniform, debt-financed tax cut has a positive present-value multiplier on consumption, stemming from intertemporal substitution by the savers, who hold the public debt.

> *If fiscal policy is used as a deliberate instrument for the more equal distribution of incomes, its effect in increasing the propensity to consume is, of course, all the greater.* (Keynes, 1936, Book III, Chapter 8, Section II).

The aftermath of the Great Recession has revived a classic debate on the effects of so-called fiscal stimulus programmes. This debate has often focused on the role of government debt. Less prominent in the debate is the fact that the rise in public debt in many countries has ensued from stimulus packages that have taken the form of transfers to specific income groups, rather than purchases of goods and services (Giambattista and Pennings, 2011; Mehrotra, 2011; Oh and Reis, 2011). It is therefore important to understand the distributional consequences of stimulus programmes, particularly in light of the upward trajectory of public debt they often imply.

In this article, we study fiscal stimulus policies in the form of temporary tax cuts. We interpret redistribution as revenue-neutral tax cuts to a fraction of the population financed by a tax rise to another; by construction, this policy changes the lifetime income (wealth) of private agents. We interpret public debt as a form of intertemporal redistribution that does not affect the lifetime income of agents, and is by construction not revenue neutral for the government.

We conduct our analysis in a framework featuring heterogenous agents, who differ in their degree of impatience, and imperfect financial markets. This setup, sometimes

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labelled Borrower-Saver model, has become increasingly popular in the recent literature.[1] The resulting model resembles the classic savers-spenders (SS henceforth) model of fiscal policy (Mankiw, 2000) in which 'myopic' households, who merely consume their income, co-exist with standard, intertemporally optimising households.[2] Ours is a variant of the SS model in two respects: first, both agents are intertemporal maximisers – so that borrowing and lending take place in equilibrium – but a fraction of agents face a suitably defined borrowing limit; second, the distribution of debt/saving across agents is endogenous.

In this model, public debt is intimately related to redistribution. A debt-financed, one-time uniform tax cut today is equivalent to a redistribution from the agents who hold the public debt (savers) to those who do not (borrowers) today, followed by a (potentially persistent) redistribution in the opposite direction from the next period onwards, as debt is repaid. The persistence of the implicit future redistribution depends on the speed at which debt is repaid; this can generate endogenously persistent effects of purely transitory tax cuts.

As our model features credit market imperfections, it is tempting to think that Ricardian equivalence readily fails, so that (lump-sum) tax cuts produce positive (and possibly large) effects on aggregate demand. We first show that this reasoning can be misleading, because the conclusion hinges on two crucial elements:

(*i*) whether the steady-state distribution of consumption across agents is uniform; and
(*ii*) whether labour supply is endogenous

In fact, the baseline version of our model with perfectly flexible prices produces two paradoxical results. First, and despite the presence of borrowing frictions, a tax redistribution that favours the constrained agents (a tax cut to the borrowers financed by a rise in taxes to the savers) is completely neutral on aggregate consumption if either labour supply is inelastic or the steady-state distribution of consumption is uniform (e.g. if the borrowing limit is zero – as implicit in the traditional SS model – and profit income is either zero or redistributed uniformly across agents).[3]

Second, even if the steady-state distribution of consumption is not uniform (so that, e.g. a fraction of agents hold private debt and another fraction a corresponding amount of savings), a tax redistribution generates a contraction in aggregate spending.

The intuition for these results is that the steady-state distribution of consumption (and wealth) governs the (intensity of the) income effect on labour supply. When steady-state consumption levels are equalised, the income effects on the agents' individual labour supplies are symmetric. In response to a tax redistribution, borrowers

[1] See for instance Eggertsson and Krugman (2012) and Monacelli and Perotti (2012). These models are variants of the RBC-type borrower–saver framework proposed in for example, Kiyotaki and Moore (1997), and extended to a New Keynesian environment by Iacoviello (2005) and Monacelli (2010); for an early analysis see Becker (1980) and Becker and Foias (1987).

[2] The classic savers-spenders model has been extended by, among others, Galí *et al.* (2007) and Bilbiie (2008) to include nominal rigidities and other frictions to study questions ranging from the effects of government spending to monetary policy analysis and equilibrium determinacy.

[3] Throughout the article, we abstract from the accumulation of physical capital to focus on one source of failure of Ricardian equivalence: sticky prices. We hint to some of the possible implications of capital accumulation in the concluding Section.

choose to work less and savers to work more in an exactly offsetting way. When the distribution of wealth is such that, realistically, borrowing-constrained agents consume relatively less in steady state, their reduction in labour supply more than compensates the increase in labour supply by the savers, leading to an overall contraction in spending and output.

A uniform tax cut financed by issuing public debt (held by the savers), which is repaid by uniform taxation in the future amounts, *de facto*, to redistributing from savers to borrowers today and reversing that redistribution in the future (when debt is repaid). Within each period, the same logic of redistribution described above applies, so that either the redistribution is neutral or it generates paradoxical results: the tax cut today is contractionary and the tax increase tomorrow is expansionary. The key extra element is that these contradicting forces are exactly symmetric: the present-value multiplier on consumption is always zero.

A large empirical literature (Blanchard and Perotti, 2002; Romer and Romer, 2010; Favero and Giavazzi, 2012; Mertens and Ravn, 2012; Perotti, 2012) identifies tax innovations using a variety of approaches and studies their macroeconomic effects. While those studies often disagree as to the magnitude of the multipliers, they all find contractionary effects in response to positive tax shocks, which casts serious doubt on the implications of the flexible-price model summarised above.

Matters are different with nominal price rigidity and even in the case of a uniform steady-state distribution of consumption. Two elements are typical of the sticky-price environment. First, as firms cannot optimally adjust prices, the increase in borrowers' consumption ensuing from the tax cut generates an increase in labour demand. Second, the rise in the real wage that results from the expansion in labour demand generates, for one, a further income effect on borrowers and hence a further expansion in their consumption; it also results in a fall in profits, with an additional negative income effect on the savers' labour supply that is absent under flexible prices.

In this scenario, we obtain two main results. First, a revenue neutral tax redistribution is expansionary on aggregate spending, as well as inflationary. Second, a debt-financed uniform tax cut generates a current expansion in aggregate spending, followed by a contraction. Crucially, however, the two effects are not symmetric: the present-value multiplier of a debt-financed tax cut is positive regardless of how fast debt is repaid, whereas it would be zero under the same conditions if prices were flexible.

The reason why the effect of a uniform tax cut goes beyond the mere sum of its implied redistributional components (from savers to borrowers, today; and from borrowers to savers, in the future) stems from intertemporal substitution: real interest rates fall, since the future *de facto* transfer from borrowers to savers generates a fall in demand and deflation, which boosts savers' consumption today. This effect is stronger (the stronger the intertemporal substitution channel, the more flexible are prices, or the more aggressive is monetary policy), and disappears when the intertemporal-substitution channel is turned off (when there are no equilibrium fluctuations in interest rates).

In the limit, if the debt-financed uniform tax cut is repaid in the indefinite future through permanently higher (but constant) future taxes, a uniform tax cut has effects that are identical to a one-time redistribution from savers to borrowers, although the two policies are very different in nature. The reason for this equivalence stems from the key role of intertemporal substitution in shaping the effects of public debt. With debt that is repaid by permanently higher (but constant) taxation from tomorrow onwards, intertemporal substitution ceases to matter.

1. Public Debt and Redistribution with Savers and Borrowers

This section outlines our economic environment, which consists of two types of households: patient and impatient, where the latter is subject to a (fixed) borrowing constraint. Furthermore, monopolistically competitive firms produce consumption goods setting prices in a staggered fashion, a monetary authority sets nominal interest rates and a government raises lump-sum taxes (issues lump-sum transfers) and issues short-term nominal debt. We also discuss the solution method and the details of the fiscal policy experiments we consider.

1.1. *The Model*

There is a continuum of households [0,1] indexed by j, all having the same utility function

$$U(C_{j,t}, N_{j,t}) = \ln\ C_{j,t} - \chi_j \frac{N_{j,t}^{1+\varphi}}{1+\varphi},$$

where $\varphi > 0$ is the inverse of the labour supply elasticity. The agents differ in their discount factors $\beta_j \in (0,1)$ and possibly in their preference for leisure χ_j. Specifically, we assume that there are two types of agents $j = s,b$, and $\beta_s > \beta_b$.

All households (regardless of their discount factor) consume an aggregate basket of individual goods $z \in [0,1]$, with constant elasticity of substitution ε: $C_t = [\int_0^1 C_t(z)^{(\varepsilon-1)/\varepsilon} \mathrm{d}z]^{\varepsilon/(\varepsilon-1)}$, $\varepsilon > 1$. Standard demand theory implies that total demand for each good is $C_t(z) = [P_t(z)/P_t]^{-\varepsilon} C_t$, where $C_t(z)$ is total demand of good z, $P_t(z)/P_t$ its relative price and C_t aggregate consumption.[4] The aggregate price index is $P_t^{1-\varepsilon} = \int_0^1 P_t(z)^{1-\varepsilon} \mathrm{d}z$.

A $1-\lambda$ share is represented by households who are patient: we label them savers, discounting the future at β_s. Consistent with the equilibrium outcome (discussed below) that patient agents are savers (and hence will hold the bonds issued by impatient agents), we impose that patient agents also hold all the shares in firms.

Each saver chooses consumption, hours worked and asset holdings (bonds and shares), solving the standard intertemporal problem:

$$\max\ \mathbb{E}_t \left[\sum_{i=0}^{\infty} \beta_s^i U(C_{s,t+i}, N_{s,t+i}) \right],$$

[4] This equation holds in aggregate because the same static problem is solved by both types of households.

subject to the sequence of constraints:

$$C_{s,t} + B_{s,t+1} + A_{s,t+1} + \Omega_{s,t+1}V_t \leq \frac{1+I_{t-1}}{1+\Pi_t}B_{s,t} + \frac{1+I_{t-1}}{1+\Pi_t}A_{s,t} + \Omega_{s,t}(V_t + \mathcal{P}_t) + W_tN_{s,t} - \tau_{s,t}, \tag{1}$$

where $\mathbb{E}_t$ is the expectations operator, $C_{s,t}, N_{s,t}$ are consumption and hours worked by the patient agent, W_t is the real wage, $A_{s,t}$ is the real value at beginning of period t of total private assets held in period t ($1 + \Pi_t \equiv P_t/P_{t-1}$ is the gross inflation rate), a portfolio of one-period bonds issued in $t-1$ on which the household receives nominal interest I_{t-1}. V_t is the real market value at time t of shares in intermediate good firms, $\mathcal{P}_t$ are real dividend payoffs of these shares, $\Omega_{s,t}$ are share holdings, $\tau_{s,t}$ are per capita lump-sum taxes paid by the saver, and $B_{s,t}$ are the savers' holdings of nominal public bonds which deliver the same nominal interest as private bonds.

The Euler equations – for bond and share holdings, respectively – and the intratemporal optimality condition are as follows:[5]

$$C_{s,t}^{-1} = \beta_s\mathbb{E}_t\left(\frac{1+I_t}{1+\Pi_{t+1}}C_{s,t+1}^{-1}\right) \text{ and } V_t = \beta_s\mathbb{E}_t\left(\frac{C_{s,t}}{C_{s,t+1}}\frac{V_{t+1}+\mathcal{P}_{t+1}}{1+\Pi_{t+1}}\right), \tag{2}$$

$$\chi_sN_{s,t}^{\varphi} = \frac{1}{C_{s,t}}W_t. \tag{3}$$

The rest of the households on the $[0,\lambda]$ interval are impatient (and will borrow in equilibrium, hence we index them by b for borrowers) face the intertemporal constraint:

$$C_{b,t} + A_{b,t+1} \leq \frac{1+I_{t-1}}{1+\Pi_t}A_{b,t} + W_tN_{b,t} - \tau_{b,t}, \tag{4}$$

as well as the additional borrowing constraint (on borrowing in real terms) at all times t:

$$-A_{b,t+1} \leq \bar{D},$$

$$\chi_bN_{b,t}^{\varphi} = \frac{1}{C_{b,t}}W_t, \tag{5}$$

$$C_{b,t}^{-1} = \beta_b\mathbb{E}_t\left(\frac{1+I_t}{1+\Pi_{t+1}}C_{b,t+1}^{-1}\right) + \psi_t, \tag{6}$$

where ψ_t takes a positive value whenever the constraint is binding. Indeed, because of our assumption on the relative size of the discount factors, the borrowing constraint will bind in steady state (we discuss this in more detail below).

Each individual good is produced by a monopolistic competitive firm, indexed by z, using a technology given by $Y_t(z) = N_t(z)$. Cost minimisation taking the wage as given implies that real marginal cost is W_t. The profit function in real terms is given by $\mathcal{P}_t(z) = [P_t(z)/P_t]Y_t(z) - W_tN_t(z)$, which aggregated over firms gives total profits

[5] These conditions must hold along with the usual transversality conditions.

$\mathcal{P}_t = (1 - W_t\Delta_t)Y_t$. The term Δ_t is relative price dispersion defined following Woodford (2003) as $\Delta_t \equiv \int_0^1 [P_t(z)/P_t]^{-\varepsilon}\,\mathrm{d}z$.

A monetary authority sets the nominal interest rate in response to fluctuations in expected inflation (we assume for simplicity that target inflation is zero):

$$1 + I_t = \Phi(1 + \mathbb{E}_t\Pi_{t+1}),$$

where $\Phi(1) = \beta_s^{-1} > 1$.

The government issues B_{t+1} one-period bonds, which are held only by the savers. To focus on the effects of taxation and public debt, we abstract from government spending. Hence, the government budget constraint reads:

$$B_{t+1} = \left(\frac{1 + I_{t-1}}{1 + \Pi_t}\right)B_t - \tau_t, \tag{7}$$

where τ_t are total tax revenues, that is, $\tau_t = \lambda\tau_{b,t} + (1 - \lambda)\tau_{s,t}$. Note that the assumption that government spending is fixed implies that exogenous variations in taxes will readily constitute a test of whether Ricardian equivalence holds in our model.

In an equilibrium of this economy, all agents take as given prices (with the exception of monopolists who reset their good's price in a given period), as well as the evolution of exogenous processes. A rational expectations equilibrium is then as usually a sequence of processes for all prices and quantities introduced above such that the optimality conditions hold for all agents and all markets clear at any given time t. Specifically, labour market clearing requires that labour demand equal total labour supply, $N_t = \lambda N_{b,t} + (1 - \lambda)N_{s,t}$. Private debt is in zero net supply $\int_0^1 A_{j,t+1} = 0$, and hence, since agents of a certain type make symmetric decisions:

$$\lambda A_{b,t+1} + (1 - \lambda)A_{s,t+1} = 0.$$

Equity market clearing implies that share holdings of each saver are

$$\Omega_{s,t+1} = \Omega_{s,t} = \Omega = \frac{1}{1 - \lambda}.$$

Finally, by Walras' Law the goods market also clears. The resource constraint specifies that all produced output will be consumed:

$$C_t = Y_t = \frac{N_t}{\Delta_t}, \tag{8}$$

where $C_t \equiv \lambda C_{b,t} + (1 - \lambda)C_{s,t}$ is aggregate consumption and Δ_t is relative-price dispersion.

All bonds issued by the government will be held by savers. Market clearing for public debt implies:

$$(1 - \lambda)B_{s,t+1} = B_{t+1}. \tag{9}$$

In our model, fiscal policy matters only through the impact of taxes (transfers) on borrowers. Substituting (7) and (9) and the definition of total taxes in the savers' budget constraint, we obtain:

$$C_{s,t} + A_{s,t+1}\frac{1 + I_{t-1}}{1 + \Pi_t} \leq \frac{\lambda}{1 - \lambda}\tau_{b,t} + A_{s,t} + \frac{1}{1 - \lambda}\mathcal{P}_t + W_tN_{s,t}. \tag{10}$$

Savers internalise the government budget constraint through their public debt holdings and so recognise that a transfer to borrowers today effectively implies a tax on themselves, today or in the future. In this sense, public debt works as a mechanism to redistribute wealth among agents, intra and inter-temporally. The higher the fraction of borrowers, the more sensitive the consumption of savers to a change in the tax on borrowers (*ceteris paribus*).

Note that the only fiscal variable appearing in the equilibrium conditions is $\tau_{b,t}$, the level of taxes on borrowers – without any reference to the aggregate level of taxes or public debt. However, the tax process itself needs to respond to public debt to ensure sustainability – but it still matters for the aggregate allocation only through its impact on taxes on borrowers. To close the model, we need to specify how fast this adjustment takes place, and how the burden of readjustment is shared between savers and borrowers.

1.2. *Tax Rules and Equilibrium Dynamics*

We solve our model locally by log-linearising it around a zero-inflation steady state, in which the borrowing constraint always binds. To check the accuracy of this solution method, we perform a series of tests based on Den Haan's (2010) dynamic Euler equation test for different values of some key parameters, including those pertaining to shock processes.[6] To anticipate, that analysis shows that for the baseline calibration described below, the constraint keeps binding virtually all the time, and approximation errors are negligible – suggesting that our solution method is valid at least for the baseline calibration we consider.

Henceforth, a small letter denotes log-deviations of a variable from its steady-state value, with two exceptions: taxes/transfers and public debt are in deviations from

Table 1
Summary of the Log-linear Model

Euler equation, S	$\mathbb{E}_t c_{s,t+1} - c_{s,t} = i_t - \mathbb{E}_t \pi_{t+1}$
Labour supply, S	$\varphi n_{s,t} = w_t - c_{s,t}$
Labour supply, B	$\varphi n_{b,t} = w_t - c_{b,t}$
Budget constraint, B	$\gamma c_{b,t} + \bar{D}_Y(i_{t-1} - \pi_t) = WN_b/C(w_t + n_{b,t}) - t_{b,t}$
Production function	$y_t = n_t$
Phillips curve	$\pi_t = \beta \mathbb{E}_t \pi_{t+1} + (1-\theta)(1-\theta\beta_s)/\theta/w_t$
Government debt	$\beta_s b_{t+1} = B_Y(i_{t-1} - \pi_t) + b_t - t_t$
Lump-sum taxes	$t_t = \lambda t_{b,t} + (1-\lambda) t_{s,t}$
Tax rule	$t_{j,t} = \phi_B^j b_t - \epsilon_{j,t} (j = b, s)$
Labour market clearing	$n_t = \lambda n_{b,t} + (1-\lambda) n_{s,t}$
Aggregate consumption	$c_t = \lambda\gamma c_{b,t} + (1-\lambda\gamma) c_{s,t}$
Resource constraint	$y_t = c_t$
Monetary policy	$i_t = \phi_\pi \mathbb{E}_t \pi_{t+1}$

[6] The results of these accuracy tests are reported in Appendix E; they consist loosely speaking of measuring how different the solution of our log-linearised method is from that of a method that only uses the log-linearised solution to calculate next period's behaviour, while other variables are calculated using the true, non-linear equations of the model. An important by-product of this analysis is that we obtain, for a given set of parameter values, a measure of how often the constraint stops binding. See Den Haan (2010) for further details.

steady state, as a share of steady-state output $Y(t_{j,t} \equiv (\tau_{j,t} - \tau_j)/Y; b_t \equiv (B_t - B)/Y)$ and interest and inflation rates are in absolute deviations from their steady-state values. All log-linearised equilibrium conditions are outlined in Table 1, where $B_Y \equiv B/Y$, $\bar{D}_Y \equiv \bar{D}/Y$ and we used the aggregate resource constraint rather than savers' budget constraint (by virtue of Walras' Law).

In our log-linear equilibrium, we assume a general financing scheme whereby taxes on each agent increase to repay the outstanding debt but only gradually so:

$$t_{j,t} = \phi_B^j b_t - \epsilon_{j,t}, \tag{11}$$

where $j = b,s$, and ϵ_t is an exogenous, possibly persistent stochastic process with $\mathbb{E}_t \epsilon_{t+1} = \rho \epsilon_t$, $\rho \geq 0$.

This tax rule is general enough to allow taxes on each agent to react to stabilise government debt ($\phi_B^j \geq 0$ is the debt feedback coefficient), and asymmetric changes in taxation for the two agents ($\epsilon_{j,t}$ is a random and possibly persistent innovation).

1.3. *Steady State*

We focus on a deterministic steady state where inflation is zero. As the constraint binds in steady state ($\psi = C_b^{-1}[1 - (\beta_b/\beta_s)] > 0$ whenever $\beta_s > \beta_b$), patient agents are net borrowers and steady-state private debt is $A_b = -\bar{D}$; by debt market clearing, then the patient agents are net lenders and their private bond holdings are $A_s = \lambda \bar{D}/(1 - \lambda)$.

To simplify the analysis, we make the further assumption that agents work the same number of hours in steady state: $N_b = N_s = N$. This assumption is consistent with the view that there are no wealth effects on long-run hours worked. Specifically, the relative weight of leisure in the utility function needs to be different across agents, $\chi_s \neq \chi_b$, by precisely the amount needed to make (only) steady-state hours identical across groups, $N_b = N_s = N$.

The utility weights χ_s and χ_b consistent with this assumption can be shown to be:

$$\chi_s = \frac{1}{N^{1+\varphi}\left(\frac{1}{1+\mu} + \frac{1}{1-\lambda}\frac{\mu}{1+\mu} + \frac{\lambda}{1-\lambda}R\bar{D}_Y\right)} < \chi_b = \frac{1}{N^{1+\varphi}\left(\frac{1}{1+\mu} - R\bar{D}_Y\right)}, \tag{12}$$

where R is the net real interest rate obtained from the Euler equation of savers, $R = I = \beta_s^{-1} - 1$ and $\mu \geq 0$ is the steady-state net markup.

The second equation in (12) determines N as a function of χ_b, and the first determines the χ_s that delivers the equalisation of hours. Note that $\chi_s < \chi_b$ (to work the same steady-state hours, savers need to dislike labour less).

The per-group steady-state shares of consumption in total consumption are

$$\frac{C_b}{C} \equiv \gamma = \frac{1}{1+\mu} - R\bar{D}_Y \leq 1,$$
$$\frac{C_s}{C} = \frac{1-\lambda\gamma}{1-\lambda} \geq 1.$$

Note that in the particular case of $\mu = 0$ and zero private debt limit, $\bar{D}_Y = 0$, we have $\gamma = 1$, implying that the distribution of steady-state consumption is uniform, $C_b = C_s = C$.

1.4. *Two Special Cases*

In the remainder of this article, we focus on two fiscal policy arrangements that allow us to obtain analytical solutions:

(*i*) pure redistribution and
(*ii*) a debt-financed tax cut.

1.4.1. *Pure redistribution (Robin Hood)*

Consider first a transfer that takes place within the period, so that the budget is balanced every period:

$$t_{b,t} = -\epsilon_t, t_{s,t} = \frac{\lambda}{1-\lambda}\epsilon_t. \tag{13}$$

In this scenario, $t_{b,t}$ is exogenous. Taxes on savers adjust to ensure public debt sustainability but this is irrelevant for the allocation. This experiment is equivalent to having a pure Robin Hood policy that taxes savers and redistributes the proceedings to borrowers within the period. Importantly, such a change in taxation is revenue neutral for the government, but changes the wealth (the lifetime income) of both agents.

1.4.2. *Uniform tax cut financed with public debt*

Alternatively, consider a uniform tax cut ($t_{b,t} = t_{s,t} = t_t$) of size ϵ_t^B to both agents, financed via public debt held by the savers.

Unlike the previous experiment, this policy change is obviously not revenue neutral, but does not *per se* affect the wealth or lifetime income of agents. To see this, consider, for the sake of simplicity, the government budget constraint log-linearised around a steady state with zero public debt ($B_Y = 0$):

$$\beta_s b_{t+1} = b_t - t_t. \tag{14}$$

The aggregate tax rule $t_t = \phi_B b_t - \epsilon_t^B$ replaced in the government budget constraint (14) implies the debt accumulation equation:

$$b_{t+1} = (1-\phi_B)\beta_s^{-1} b_t + \beta_s^{-1}\epsilon_t^B. \tag{15}$$

To ensure debt sustainability, the response of taxes to debt needs to obey:[7]

$$\phi_B \in [1-\beta_s, 1]. \tag{16}$$

[7] Note, however, that condition (16) need not hold for taxation of both agents, but for the aggregate response. Indeed, it would be sufficient if for instance taxes of savers fulfilled (16), that is, $\phi_B^s > 1 - \beta_s$ and taxes of borrowers did not respond to debt at all, $\phi_B^b = 0$. When the condition is fulfilled, (15) can be solved independently of the rest of the model to determine the path of public debt; this is due to our assumption of zero public debt in steady state, which makes interest payments irrelevant to first order.

For the sake of simplicity, we assume that the tax shock has zero persistence: $\epsilon^B_{t+i} = 0$ for any $i > 0$. In that case, the debt accumulation equation implies that taxes at $t + 1$ are given by

$$\begin{aligned} t_{t+1} &= \phi_B b_{t+1} = \phi_B(1-\phi_B)\beta_s^{-1} b_t + \phi_B \beta_s^{-1}\epsilon_t^B \\ &= (1-\phi_B)\beta_s^{-1} t_t + \beta_s^{-1}\epsilon_t^B. \end{aligned} \tag{17}$$

Equation (17) shows that from the period immediately following the tax cut, the tax process follows an AR(1) process, with persistence $(1 - \phi_B)\beta_s^{-1}$ and an initial value proportional to the initial tax cut.[8] At any time $t + i$ for $i > 1$, taxes obey:

$$t_{t+i} = (1-\phi_B)^i \beta_s^{-i} t_t + (1-\phi_B)^{i-1}\beta_s^{-i}\epsilon_t^B. \tag{18}$$

Alternative values of parameter ϕ_B describe different horizons over which debt stabilisation is achieved (and therefore the initial tax cut is reversed), as well as different initial values for the size of the initial tax adjustment. It is useful to consider two extreme cases.

(*i*) *One-period debt stabilisation.* In this case, $\phi_B = 1$. A cut in taxes today ϵ_t^B implies the tax process

$$t_t^{FS} = b_t - \epsilon_t^B;\ t_{t+1}^{FS} = \beta_s^{-1}\epsilon_t^B;\ t_{t+i}^{FS} = 0 \text{ for } i > 2. \tag{19}$$

The tax process lives for only one period, as all debt is repaid in the next period. Therefore, the tax adjustment in period $t + 1$ is *a fortiori* the largest in this case. Recall that, as taxation is uniform but all debt is held by the savers, this experiment is equivalent to a redistribution at time t of amount ϵ_t^B from the savers to the borrowers, followed by a reverse transfer in period $t + 1$ of $\beta_s^{-1}\epsilon_t^B$. Effectively, the government lends to the borrowers.

(*ii*) *No debt stabilisation.* At the other extreme, we have $\phi_B = 1 - \beta_s$. This implies that the tax process has a unit root:

$$\begin{aligned} t_t^{NS} &= (1-\beta_s) b_t - \epsilon_t^B, \\ t_{t+i}^{NS} &= t_t^{NS} + \beta_s^{-1}\epsilon_t^B \text{ for any } i > 1. \end{aligned} \tag{20}$$

The increase in taxes from period $t + 1$ onwards is the longest in this case: taxes increase for the indefinite future by the constant amount $(\beta_s^{-1} - 1)\epsilon_t^B$. This experiment amounts to a redistribution from savers to borrowers of size ϵ_t^B followed by a permanent transfer of $(\beta_s^{-1} - 1)\epsilon_t^B$ from borrowers to savers from $t + 1$ onwards.

As the parameter ϕ_B increases, the persistence of the tax process diminishes (the initial tax cut is repaid faster) and (hence) the adjustment in taxes in period $t + 1$

[8] If we did not restrict the tax-cut shock to last for only one period, taxes would follow an ARMA(1,1) process: $t_{t+1} = (1 - \phi_B)\beta_s^{-1} t_t + \beta_s^{-1}\epsilon_t^B - \epsilon_{t+1}^B$.

becomes larger, as the present discounted value of taxes needs to be just enough to ensure repayment of the initial debt.[9]

2. Flexible Prices and Ricardian Equivalence

We begin by assuming that prices are fully flexible. We show that in an environment in which the steady-state levels of consumption of borrowers and savers are different, Ricardian equivalence fails: changes in lump-sum taxes affect the real allocation. However, the predictions concerning the effect of tax cuts are counter-intuitive and contrary to empirical findings – which motivates our further analysis of other deviations from Ricardian equivalence.

Log-linearising (4) and (5) around the steady-state, and combining, we obtain:

$$\begin{aligned} n_{b,t} &= \frac{\gamma(1+\mu)-1}{\varphi\gamma(1+\mu)+1} w_t + \frac{1+\mu}{\varphi\gamma(1+\mu)+1}(\bar{D}_Y r_{t-1} + t_{b,t}), \\ c_{b,t} &= \frac{1+\varphi}{\varphi\gamma(1+\mu)+1} w_t - \frac{\varphi(1+\mu)}{\varphi\gamma(1+\mu)+1}(\bar{D}_Y r_{t-1} + t_{b,t}), \end{aligned} \tag{21}$$

where $r_{t-1} \equiv i_{t-1} - \pi_t$.

Starting from the steady state, and in response to an increase in taxation, borrowers' hours worked decrease (in equilibrium) with the real wage because of a positive income effect (which disappears when the debt limit is zero and $\gamma(1+\mu) = 1$) and increase with taxes and interest payments.

Denote with a star a variable under flexible prices. Evaluating (21) at flexible prices (i.e. constant real marginal cost $w_t^* = 0$), replacing into the aggregate consumption definition, solving for savers' consumption at flexible prices and, using (5), we obtain the following expression for aggregate consumption (output) under flexible prices:[10]

$$c_t^* = \zeta(\bar{D}_Y r_{t-1} + t_{b,t}), \tag{22}$$

where

$$\zeta \equiv \frac{\lambda(1-\gamma)}{1-\lambda+\varphi(1-\lambda\gamma)} \frac{\varphi(1+\mu)}{\varphi\gamma(1+\mu)+1} \geq 0.$$

Equation (22) contains a reduced form expression for aggregate consumption as a function of the exogenous tax process for borrowers, $t_{b,t}$, and the predetermined real interest rate, r_{t-1}. Direct inspection of (22) in the case $\gamma = 1$ (equal steady-state consumption shares) or $\phi \to \infty$ (inelastic labour supply) suggests Proposition 1.

PROPOSITION 1. *When either labour supply is inelastic or steady-state consumption of savers and borrowers are equal, Ricardian equivalence holds – regardless of how high the fraction of borrowers λ and how tight the debt constraint $\bar{D}_Y$ are.*

The intuition for Ricardian equivalence in the two cases covered by the proposition is simple. When labour supply is inelastic, total consumption trivially equals total

[9] It can be easily shown that $\sum_{i=0}^{\infty} \beta_s^i t_{t+i} = b_t$ simply by using (18).

[10] Using also aggregate hours and the equilibrium expression for the hours of the borrower, as well as goods market clearing $c_t = n_t$.

endowment, regardless of how that endowment is distributed. When instead labour supply is elastic but steady-state consumption levels are equalised, income effects on agents' individual labour supplies (effects which are governed precisely by the steady-state consumption levels) are fully symmetric; to take one example that we elaborate on below: in response to an increase in their taxes $t_{b,t}$, borrowers want to work exactly as many hours more as savers are willing to work less when their taxes fall to balance the budget ($t_{s,t} = -\lambda(1-\lambda)^{-1}t_{b,t}$).[11]

This symmetry breaks up when steady-state consumption levels are different. In the more general case $\gamma < 1$, three features of the solution are worth emphasising.

First, Ricardian equivalence fails: any given change in lump-sum taxes on borrowers produces an effect on aggregate consumption.

Second, with $\zeta > 0$, the effect on aggregate spending is paradoxical: a rise (fall) in taxes generates a rise (fall) in consumption.

Third, even when the debt limit is zero ($\bar{D}_Y = 0$), there is still steady-state consumption inequality and Ricardian Equivalence still fails. To better understand the effects of redistribution and public debt under flexible prices, consider in turn the two extreme fiscal policy experiments described above, assuming for simplicity that $\bar{D}_Y = 0$.

2.1. *Pure Redistribution (Robin Hood)*

Consider the first policy experiment outlined in subsection 1.4 above: a within-period, balanced-budget transfer to borrowers financed by taxes on savers (13).

Replacing (13) by (22), and assuming $\bar{D}_Y = 0$, the multiplier of the tax cut on consumption reads:

$$c_t^* = -\zeta\epsilon_t < 0. \tag{23}$$

Hence, if $\gamma < 1$, consumption, output and labour hours fall: redistributing within the period from the unconstrained to the constrained agents produces a contractionary effect on aggregate activity. The intuition for this, somehow paradoxical, result is simple: the negative income effect on savers resulting from the tax redistribution is larger in absolute value than the positive income effect on borrowers.

2.2. *Public Debt*

Consider next a temporary uniform tax cut of size ϵ_t^B to both agents, financed via public debt. Under this experiment, (22), combined with the aggregate tax rule $t_t = \phi_B b_t - \epsilon_t^B$, implies that aggregate consumption obeys (assuming $\bar{D}_Y = 0$ for simplicity) for any $i > 0$:

$$c_{t+i}^* = \zeta t_{t+i}.$$

As $\zeta > 0$, the prediction under flexible prices is once again that tax cuts cause a contraction in aggregate consumption on impact. Moreover, as taxes increase in the

[11] Guerrieri and Lorenzoni (2012) discuss the relevance of group-type labour supply effects to generate an aggregate recession in a Bewley-type economy in response to a credit supply (or private 'deleverage') shock.

future in order to repay public debt even when the shock is purely transitory, the model also predicts that future consumption will increase along with future taxes.

To summarise, the implications of the model under flexible prices are inconsistent with a large empirical literature documenting that positive tax shocks are contractionary, rather than expansionary (Romer and Romer, 2010; Perotti, 2012).

The reason why tax increases are expansionary in our model is strictly related to each agent's income effect on labour supply: the income effect on savers' labour supply deriving from any given tax change is larger than that on borrowers' labour supply. Therefore, in response to a change of equal size (but of opposite sign) in their taxes, savers wish to increase their labour input more than borrowers want to decrease it.

However, it is worth noticing that the present-value multiplier on aggregate consumption is zero. The present-value multiplier of a debt-financed tax cut can be written:

$$\mathcal{M}^{*}_{debt} \equiv \frac{\partial\left(\sum_{i=0}^{\infty} \beta_s^i c^{*}_{t+i}\right)}{\partial \epsilon_t^B} = \zeta \sum_{i=0}^{\infty} \beta_s^i \frac{\partial t_{t+i}}{\partial \epsilon_t^B}$$
$$= -\zeta + \zeta \phi_B \sum_{i=1}^{\infty} (1-\phi_B)^{i-1} = 0.$$

The contractionary effects of tax cuts and the expansionary effects of future tax increases sum up to a zero net effect on the present discounted value of consumption and hours worked, regardless of how persistent public debt is. These paradoxical effects of lump-sum tax changes on aggregate consumption under flexible prices motivate our further analysis, which consists of studying a model in which price adjustment is imperfect.

3. Sticky Prices

We assume a standard Calvo (1983)-Yun (1996) monopolistic competitive environment in which intermediate good firms adjust their prices infrequently. Savers (who in equilibrium will hold all the shares in firms) maximise the discounted sum of future nominal profits.

In the following, we assume that steady-state consumption shares are equalised. This is achieved by assuming that both the debt limit and steady-state profits are zero; the latter in turn is obtained with a sales subsidy $\sigma = \mu$, so that profits' share in total output is zero. Note also that under this assumption, the implied weights on leisure in the utility function are equal across agents.[12]

The steady-state symmetry of consumption levels makes aggregation simple, and allows us to isolate the role of sticky prices in generating a failure of Ricardian equivalence. Under these conditions, the aggregate constant-consumption labour supply curve has the same parameters as the individual ones: $\varphi n_t = w_t - c_t$, implying $w_t = (1+\varphi)c_t$. Replacing these equations in the definition of aggregate consump-

[12] The alternative way to achieve this outcome would be to assume that there are steady-state transfers that redistribute asset income evenly; the assumption we use has the relative merit of being consistent with evidence pointing to the long-run share of pure economic profits being virtually zero. We also avoid taking a stand on the amount of steady-state redistribution through lump-sum transfers, which is very hard to measure.

tion, solving for consumption of savers, and substituting in the savers' Euler equation, we obtain the aggregate demand equation:

$$c_t = \mathbb{E}_t c_{t+1} - \delta^{-1}(i_t - \mathbb{E}_t \pi_{t+1}) - \delta^{-1}\eta(t_{b,t} - \mathbb{E}_t t_{b,t+1}),$$
$$\text{where } \delta \equiv 1 - \frac{\lambda\varphi}{1-\lambda} \text{ and } \eta \equiv \frac{\lambda}{1-\lambda}\frac{\varphi}{1+\varphi}. \tag{24}$$

Bilbiie (2008) shows that – in a model that is equivalent to ours with $\bar{D}_Y = 0$ – for values of $\lambda > 1/(1+\phi)$, δ becomes negative: the aggregate elasticity of intertemporal substitution changes sign and interest rate cuts become contractionary. In that 'inverted aggregate demand logic' region, the monetary policy rule needs to follow an inverted Taylor principle to ensure determinacy and rule out sunspot fluctuations. In the remainder of this article, we focus on parameter values that imply that $\delta > 0$ so that standard aggregate demand logic holds.[13]

Finally, our Calvo-Yun environment implies a standard forward-looking Phillips curve

$$\pi_t = \beta\mathbb{E}_t\pi_{t+1} + \kappa c_t, \text{ where } \kappa \equiv (1+\varphi)\frac{(1-\beta_s\theta)(1-\theta)}{\theta} \tag{25}$$

with $\theta \in [0,1]$ being the probability that each intermediate producer keeps its price constant in every period.

The model is closed by the following Taylor-type interest rate rule:[14]

$$i_t = \phi_\pi \mathbb{E}_t \pi_{t+1}, \tag{26}$$

where $\phi_\pi > 1$.

3.1. *Pure Redistribution (Robin Hood)*

Consider once again the effect of pure redistribution – a transfer ϵ_t to borrowers financed by taxes on savers within the period. The tax processes are once again given by (13). It is instructive to simplify even further and first consider the case where the shock lasts only one period, $\mathbb{E}_t\epsilon_{t+1} = 0$.

As the model is entirely forward-looking, expected values of consumption and inflation are also zero: $\mathbb{E}_t c_{t+1} = \mathbb{E}_t \pi_{t+1} = 0$, and the solution is simply:

$$c_t = -\delta^{-1}\eta t_{b,t} = \mathcal{M}_{red}\epsilon_t,$$
$$\pi_t = -\kappa\delta^{-1}\eta t_{b,t} = \kappa\mathcal{M}_{red}\epsilon_t,$$
$$\text{where } \mathcal{M}_{red} \equiv \frac{\lambda}{1-\lambda(1+\varphi)}\frac{\varphi}{1+\varphi}$$

[13] In our framework with a non-zero debt limit, this result will depend on the value of the debt limit $\bar{D}_Y$ (intuitively, even when $\delta < 0$ an increase in the real rate needs not necessarily be expansionary, because of the contractionary effect on aggregate demand of interest payments on outstanding debt). But the same intuition holds, in that for values of the share of borrowers above that threshold, the effects of the type of fiscal shocks analysed here are overturned.

[14] In the Appendix, we show that our results are largely robust to considering a Taylor rule that reacts to current, realised inflation π_t.

is the consumption multiplier of redistribution under sticky prices. These expressions suggest Proposition 2.

PROPOSITION 2. *A within-the-period revenue-neutral transitory redistribution from savers to borrowers generates an expansion in aggregate consumption and inflation, as long as the elasticity of aggregate demand to interest rate is negative* $(\delta > 0)$, *that is*

$$\lambda < \frac{1}{1+\varphi}.$$

To understand the intuition, recall first what happens under flexible prices, if income effects on both agents are equal (they have the same long-run consumption values): labour supply of the borrowers shifts downwards but labour supply of the savers shifts upwards by the same amount. Labour demand does not change either – so redistribution has no effect.

With sticky prices and even in the knife-edge case of uniform steady-state consumption, two key ingredients break this neutrality. First, recall that output is demand-determined; the increase in borrowers' consumption generates a demand effect: labour demand increases as some firms are stuck with the old, suboptimally low price. The second key ingredient is the asymmetry between income effects. Faced with an increase in the real wage (marginal cost), the savers recognise that they face an extra negative income effect (that is absent with flexible prices) as their profit income falls. In equilibrium, they will therefore work more than the borrowers are willing to work less, therefore supporting the aggregate expansion in consumption. This income effect is increasing in the fraction of borrowers and decreasing with labour supply elasticity but only up to the threshold given in the Proposition.[15]

In the more general case when redistribution is persistent (exogenously), with $\mathbb{E}_t\epsilon_{t+1} = \rho\epsilon_t$, the responses of inflation and consumption to taxes on borrowers are reported in Proposition 3 (a proof of which can be found in the Appendix).

PROPOSITION 3. *In response to an exogenously persistent redistribution from savers to borrowers* ϵ_t, *inflation and consumption (output) follow:*

$$\pi_t = \frac{\beta^{-1}\kappa\delta^{-1}\eta(1-\rho)}{\det}\epsilon_t$$
$$c_t = \frac{\delta^{-1}\eta(1-\rho)(\beta^{-1}-\rho)}{\det}\epsilon_t,$$

where $\det = (1-\rho)(\beta^{-1}-\rho) + \beta^{-1}\kappa\delta^{-1}(\phi_\pi - 1)\rho.$

[15] Beyond that threshold (if there are 'too many' borrowers or if labour supply is 'too inelastic') the effects of all shocks are overturned: the slope of the aggregate investment/savings (IS) curve changes sign and interest rate increases become expansionary. See Bilbiie (2008) and Bilbiie and Straub (2011) for a detailed analysis.

Qualitatively, the responses are the same as above: inflation and consumption increase when there is an exogenous tax cut to borrowers. Quantitatively, it can be easily shown that the multiplier on consumption is decreasing with the persistence parameter ρ (implying that it is always lower than the multiplier derived under zero persistence). This happens because a persistent shock generates two effects. First, it raises expected future inflation and hence – via the monetary policy rule – the real interest rate, which works to reduce savers' consumption through intertemporal substitution. Second, it enhances the negative wealth effect on labour supply by the savers, inducing them, at the margin, to reduce consumption further.[16]

3.2. *Public Debt*

We next turn to the effects of a uniform tax cut financed via issuing public debt, focusing on purely transitory shocks ($\mathbb{E}_t \epsilon^B_{t+1} = 0$) in order to isolate the role of the endogenous propagation of this tax shock through the debt accumulation process.

Debt is issued to finance the tax cut of ϵ^B_t and is repaid gradually through uniform taxes in the future; therefore, although the tax cut itself is purely transitory, its effect can be potentially long-lived because of debt accumulation and its persistence and magnitude are governed by the feedback coefficient ϕ_B.

To illustrate the mechanism, it is useful to consider the two extreme scenarios described in subsection 1.4, corresponding, respectively, to $\phi_B = 1$ and $\phi_B = 1 - \beta_s$.

3.2.1. *Full debt stabilisation*

In the case $\phi_B = 1$, a tax cut today is debt-financed and fully repaid in the next period: this amounts to a redistribution from savers to borrowers today, and from borrowers to savers in the next period.

Replacing the tax process (19) in the demand curve (24), we obtain:

$$c_t = \mathbb{E}_t c_{t+1} - \delta^{-1}(i_t - \mathbb{E}_t \pi_{t+1}) - \delta^{-1}\eta b_t + \delta^{-1}\eta(1 + \beta_s^{-1})\epsilon^B_t. \tag{27}$$

Using (27), one can derive the solutions for consumption and inflation, which are shown in Proposition 4 (the proof can be found in the Appendix).

PROPOSITION 4. *In response to a tax cut financed by public debt which is fully repaid next period by uniform taxation, consumption and inflation are given by*

$$\begin{aligned} c_t &= \delta^{-1}\eta[1 + \delta^{-1}\kappa(\phi_\pi - 1)\beta_s^{-1}]\epsilon^B_t - \delta^{-1}\eta b_t, \\ \pi_t &= \kappa^2\delta^{-2}\eta(\phi_\pi - 1)\beta_s^{-1}\epsilon^B_t - \kappa\delta^{-1}\eta b_t, \\ c_{t+1} &= -\delta^{-1}\eta\beta_s^{-1}\epsilon^B_t, \\ \pi_{t+1} &= -\kappa\delta^{-1}\eta\beta_s^{-1}\epsilon^B_t, \\ \textit{and}\, \pi_{t+i} &= c_{t+i} = 0\, \forall i \geq 2. \end{aligned}$$

[16] Finally, the effects are only slightly different if the monetary authority responds to variations in realised, rather than expected inflation. We show in the Appendix that the effects of redistribution are in that case dampened; intuitively, in response to today's inflation due to the demand effect, the monetary authority increases the real interest rate which makes savers cut consumption today by intertemporal substitution.

The foregoing results can be explained intuitively as follows. Consider, to start with, the equilibrium values obtained in period $t + 1$: in present-value terms, they are equal (but of opposite sign) to the responses of consumption and inflation to a pure redistribution described in subsection 3.1, precisely because our experiment is akin to a (reverse) redistribution from borrowers to savers in period $t + 1$.

In period t, however, we have two effects. First, the usual effect of redistribution on consumption, summarised by the term $\delta^{-1}\eta(\epsilon_t^B - b_t)$; second, an additional effect equal to $\delta^{-2}\eta\kappa(\phi_\pi - 1)\beta_s^{-1}\epsilon_t^B$, that is driven by intertemporal substitution by the savers and can be explained as follows.

In period $t + 1$, firms are faced with lower demand (due to the reverse redistribution from borrowers to savers) and cut prices, creating deflation. Savers react to this by not changing their consumption at all: the real interest rate does not move as expected inflation at $t + 2$ is zero.[17] At time t, the expected deflation implies a cut in the *ex ante* real interest rate today and (as tomorrow's consumption is unchanged) an increase in savers' consumption today – once more, by intertemporal substitution. Finally, in equilibrium firms correctly anticipate lower demand in the future and increase prices today by less than they would if redistribution were not 'reversed' in the future.

Note that consistent with this intuition, this reinforcing effect under public debt disappears when either prices are fixed ($\theta \rightarrow 1$) or there are no savers, and hence no intertemporal substitution ($\lambda = 1$, which implies $\delta^{-1} \rightarrow 0$), or no endogenous movements in real interest rates ($\phi_\pi = 1$); finally, the effect also disappears when there are no borrowers ($\lambda = 0 \rightarrow \eta = 0$), consistently with Ricardian equivalence.

The above results allow us to compute the present-value aggregate consumption multiplier of a debt-financed tax cut in the full-stabilisation case, $\mathcal{M}_{debt}^{FS}$:

$$\mathcal{M}_{debt}^{FS} \equiv \frac{\partial(c_t + \beta_s c_{t+1})}{\partial \epsilon_t^B} = \delta^{-2}\eta\kappa(\phi_\pi - 1)\beta_s^{-1} > 0, \tag{28}$$

where there is discounting at the steady-state real interest rate (which is determined by the savers' discount factor). Equation (28) immediately implies that the present-value multiplier of public debt is higher under sticky relative to flexible prices:

$$\mathcal{M}_{debt}^{FS} > \mathcal{M}_{debt}^{*} = 0.$$

Moreover, (28) shows that $\mathcal{M}_{debt}$ is identical to the intertemporal effect described previously, that is, to the component of the period-t consumption multiplier of a uniform tax cut that is over and above the multiplier due to pure redistribution, and is due to intertemporal substitution.

We can assess the magnitude of $\mathcal{M}_{debt}$ by looking at a parameterisation that is standard in the literature, namely: unitary inverse Frisch elasticity $\phi = 1$, average price duration of 1 year ($\theta = 0.75$), steady-state markup of 0.2 ($\varepsilon = 6$) and discount factor of savers $\beta_s = 0.99$.

Figure 1 plots the value of $\mathcal{M}_{debt}$ under this parameterisation, for the whole range of the share of borrowers λ for which the elasticity of aggregate demand to the interest rate is positive ($\delta > 0$), namely $\lambda < 0.5$. We consider two values of the inflation elasticity

[17] Matters are different when the monetary authority responds to realised, rather than expected inflation, but without affecting the conclusion qualitatively; we discuss this further below.

of interest rates: $\phi_\pi = 1.5$ (red dashed line) and $\phi_\pi = 3$ (blue solid line), respectively; consistent with our analytical results and intuition, the multiplier is uniformly larger for the higher value of ϕ_π. At low values of λ, until about 0.4, the multiplier is very small, below 1%. But when approaching the threshold beyond which the economy moves to the 'inverted' region, the multiplier becomes very large.[18]

3.2.2. *No debt stabilisation*

When ϕ_B tends to its lower bound given by $1 - \beta_s$, the effects of a debt-financed uniform tax cut are almost identical to the effects of pure redistribution. The intuition for this result is simple: when debt repayments are pushed into the far future, savers fully internalise the government budget constraint; taxation in the future is, for them, equivalent to taxation today. But for the borrowers, a tax cut today is disposable income. Therefore, a uniform tax cut becomes equivalent to a pure redistribution within the period when the uniform tax cut is financed with very persistent debt.

Formally, this can be seen by replacing the tax process (20) in the IS curve (24) and first noticing that, since the path of taxes from period $t + 1$ onwards is constant ((20) implies that $t_{b,t+i} = \mathbb{E}_t t_{b,t+i+1}$ for $i > 1$), all variables go back to steady state in period $t + 1$:[19] Therefore, the tax cut only has an effect at time t, when the IS curve is

$$c_t = \mathbb{E}_t c_{t+1} - \delta^{-1}(i_t - \mathbb{E}_t \pi_{t+1}) + \delta^{-1} \eta \beta_s^{-1} \epsilon_t^B,$$

which is the same as the Euler equation (24), obtained for a (purely transitory) redistribution shock of $t_{b,t} = -\beta_s^{-1} \epsilon_t^B$. Therefore, the effects of this policy are exactly

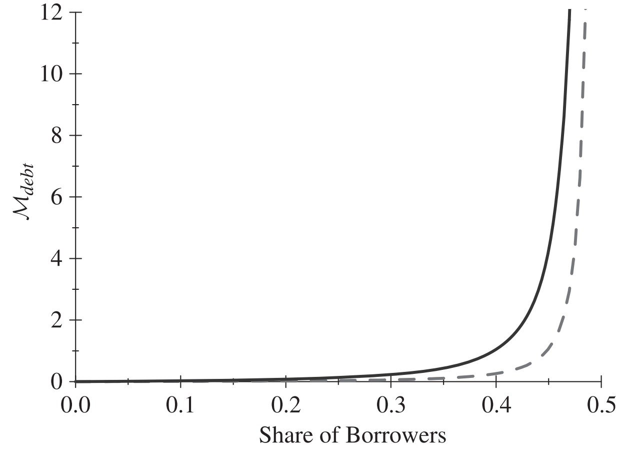

Fig. 1. *Present-Value Multiplier, for $\phi_\pi = 1.5$(Dashed Line) and $\phi_\pi = 3$(Solid Line)*

[18] For instance, under $\phi_\pi = 3$, it is about 4% when $\lambda = 0.45$ and about 12% when $\lambda = 0.47$. The reason for this abrupt increase is that the elasticity of aggregate demand to real interest rates δ^{-1} approaches infinity when λ approaches that threshold value; see Bilbiie (2008) for an elaboration of that point and Bilbiie and Straub (2004) and Bilbiie, Meier and Müller (2008) for an analysis of some fiscal policy implications.

[19] Specifically, for any $i > 0$ the IS curve becomes: $c_{t+i} = \mathbb{E}_{t+i} c_{t+i+1} - \delta^{-1}(\phi_\pi - 1)\mathbb{E}_t \pi_{t+i+1}$, which together with the Phillips curve implies the unique solution $c_{t+i} = \pi_{t+i} = 0$.

identical to those obtained in subsection 3.1 when the size of the redistribution is $\beta_s^{-1}\epsilon_t^B$, including the present-value multiplier:

$$\mathcal{M}_{debt}^{NS} = \delta^{-1}\eta\beta_s^{-1} = \beta_s^{-1}\mathcal{M}_{red}.$$

It is important to note that the two policy experiments are very different in nature: one – redistribution – changes the present discounted value of income for both agents, while the other the other – uniform tax cut – does not. Yet, they have identical effects, because what is important for propagation is the intertemporal substitution induced by changes in taxation over time. With permanently higher taxes from tomorrow onwards, there is no such intertemporal substitution and the only force at work is redistribution today.

3.2.3. *Endogenously persistent debt*

When ϕ_B takes on intermediate values – so that debt stabilisation is neither immediate nor postponed into the far future – the interplay of income effects, intertemporal substitution by savers and the demand effect due to sticky prices generate different responses that feature endogenous persistence.

To solve the model in this more general case, we exploit our previous intuition that a uniform tax cut financed by persistent debt can be reinterpreted as a transfer from savers (the holders of the debt used to finance the tax cut) to borrowers in the period when the tax cut takes place, followed by a – possibly persistent – transfer from borrowers to savers from next period onwards, when debt is being repaid. The model solution from period $t + 1$ onwards hence closely resembles the solution under a persistent transfer outlined in Proposition 3, while the solution at time t (when policy is implemented) mirrors that of Proposition 4. The full solution of the model is outlined in the Appendix, and Proposition 5 emphasises the present-value multiplier on consumption.

Proposition 5. *In response to a one-time uniform tax cut ϵ_t^B financed by issuing public debt, the present-value consumption multiplier is*

$$\mathcal{M}_{debt} \equiv \frac{\partial\left(\sum_{i=0}^{\infty}\beta_s^i c_{t+i}\right)}{\partial\epsilon_t^B} = \frac{\delta^{-2}\eta\beta_s^{-1}\kappa(\phi_\pi - 1)}{\phi_B[1-(1-\phi_B)\beta_s^{-1}] + (1-\phi_B)\beta_s^{-1}\delta^{-1}\kappa(\phi_\pi - 1)} > 0.$$

Note, to start with, that this solution nests the particular cases of full and no debt stabilisation, respectively, when $\phi_B = 1$ and $\phi_B = 1 - \beta_s$.

The key finding is that the present-value multiplier of debt is positive, and hence larger than the one under flexible prices, regardless of the value of ϕ_B satisfying (16).[20] The intuition for this is similar to the one outlined above in the extreme case of full debt stabilisation: the effects of debt go beyond the mere sum of the implied intertemporal redistributions, through intertemporal substitution generated by the movements in the real interest rate. The expectation of a future deflation triggered by

[20] This holds as long as we restrict attention to the 'standard' region whereby $\delta > 0$, and the Taylor principle is satisfied ($\phi_\pi > 1$).

the *de facto* reversal of the transfer in the future induces a fall in the long-run real interest rate today and hence boosts savers' consumption today by intertemporal substitution. Consistent with this intuition, the multiplier collapses to zero when the intertemporal-substitution channel is shut off (i.e. when prices are fixed and $\kappa = 0$ or $\phi_\pi = 1$). Furthermore, the stronger is this intertemporal substitution channel (i.e. the less sticky are prices – the higher κ – or the more aggressive is monetary policy – the higher is ϕ_π), the larger is the multiplier.[21] Finally, the intertemporal substitution channel becomes irrelevant when there is no debt stabilisation, for in that case there is in fact no intertemporal substitution: the expansionary effect of the tax cut is due solely to the redistribution from savers to borrowers today.

While the present-value multiplier is positive regardless of the speed of debt repayment ϕ_B (as long as (16) holds), its magnitude depends non-trivially on this parameter. It can be shown that for plausible calibrations (namely if $\phi_\pi < 1 + \delta\kappa^{-1}$, which is 11. 754 under our baseline calibration), the present-value multiplier is larger under 'no stabilisation' than under 'full stabilisation'. The intuition is that in those cases (if monetary policy is not too aggressive, prices are sticky enough) the intertemporal substitution channel present when debt is repaid more abruptly is weaker than the more direct expansionary effect of a redistribution today.

Figure 2 illustrates these findings by plotting the responses of consumption, inflation and public debt to a purely transitory uniform tax cut – for the baseline parameterisation described above – under three scenarios. The solid line corresponds to the 'full

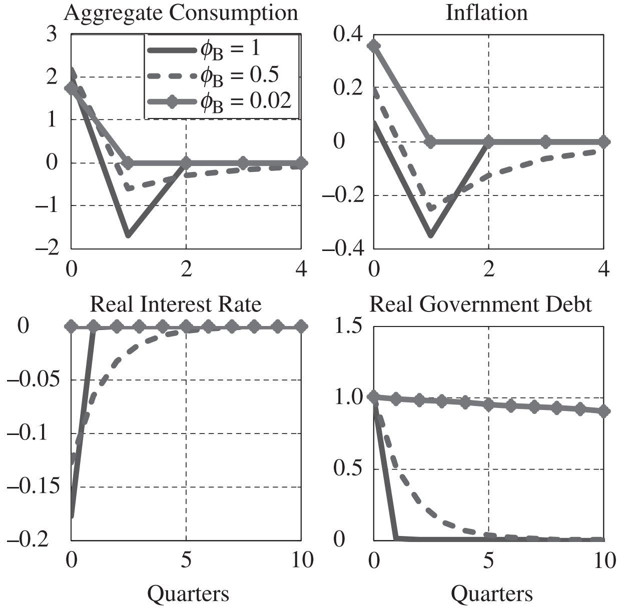

Fig. 2. *Impulse Responses to a Debt-Financed Uniform Tax Cut*

[21] It is easy to show that $\mathcal{M}_{debt}$ is increasing in both κ and ϕ_π, as long as there is some debt stabilisation $\phi_B > 1 - \beta_s$.

debt stabilisation' scenario, and the squared line is close to the 'no debt stabilisation' scenario, both of which are explained at length above. The dashed line plots the responses obtained for $\phi_B = 0.5$. The fall in consumption and inflation in the second period is smaller that under full debt stabilisation, but the recession and deflation last longer. The intuition for these intermediary values is similar to that obtained under full stabilisation. The only differences are that future transfers from borrowers to savers last longer themselves (recall the effects derived for persistent redistribution), and the initial implicit transfer of period $t + 1$ is lower (recall the discussion of (18)).

Alhough the results above were derived under the special assumptions that the private debt limit is zero, and steady-state public debt is also zero, we emphasise that they are robust to relaxing those assumptions. The reason is that the main difference, when relaxing either of those assumptions, is related to interest payments – on either private or public debt, respectively – which turn out to be quantitatively negligible. Results for these, and other, robustness experiments are available in Appendix F.

4. Conclusions

This article contributes to a vast literature studying the effects of public debt; see Elmendorf and Mankiw (1999) for a survey. The novel element is that our analysis relates public debt to another, hitherto mostly disconnected issue: redistribution through fiscal policy, in a model with heterogenous agents. The origins of the anaysis of fiscal policy redistribution can be traced at least back to Keynes (1936), who regarded it as one of the main determinants – at least of equal importance to interest rates – of the marginal propensity to consume, which was in turn the key determinant of fiscal policy multipliers.[22]

In our economy with financial imperfections, somewhat surprisingly, Ricardian equivalence holds when prices are flexible and either labour supply is inelastic, or it is elastic but the steady-state (or initial) consumption distribution is uniform. Income effects in that setup are symmetric, so one agent's decision to consume less (and, if labour is elastic, work more) is exactly compensated by another agent's decision to consume more (and work less). When the steady-state distribution of consumption is not uniform, Ricardian equivalence does fail but the effects of changes in lump-sum taxes are paradoxical when judged against the findings of a large empirical literature reviewed above: both a Robin Hood redistribution that favours the constrained borrowers and a uniform tax cut financed with public debt are contractionary. Key to this result is the asymmetry in the group-type income effects on labour supply. However, the present-value multiplier of public debt on consumption is always zero.

[22] 'The principal objective factors which influence the propensity to consume appear to be the following: [...] 5. Changes in fiscal policy. In so far as the inducement to the individual to save depends on the future return which he expects, it clearly depends [...] on the fiscal policy of the Government. Income taxes, especially when they discriminate against 'unearned' income [...] are as relevant as the rate of interest; whilst the range of possible changes in fiscal policy may be greater, in expectation at least, than for the rate of interest itself. If fiscal policy is used as a deliberate instrument for the more equal distribution of incomes, its effect in increasing the propensity to consume is, of course, all the greater'. (Keynes, 1936, Book III, Chapter 8, Section II).

Under sticky prices, lump-sum tax policies are never neutral, even when the steady-state distribution of consumption is uniform. In this environment, a Robin Hood redistribution that favours the borrowers is expansionary on aggregate activity. Unlike under flexible prices, a uniform tax cut financed with public debt has a positive present-value multiplier effect on consumption. In other words, the effects of debt go beyond the mere sum of the implicit intertemporal redistributions (from savers to borrowers today, and *vice versa* from tomorrow onwards). Key to this result is intertemporal substitution by the savers: the perspective of a deflationary recession tomorrow triggers a fall in interest rate today, boosting savers' consumption today. For this reason, and although the policy change has no wealth effect *per se* (it does not change the lifetime income of agents), the present-value multiplier on consumption is positive.

The finding that the present-value multiplier is positive holds regardless of how quickly debt is repaid – although the speed of debt stabilisation does influence the magnitude of the multiplier – as long as it is indeed repaid. In the limit, when the tax cut is financed by permanently higher taxes from next period onwards (a scenario we label 'no debt stabilisation'), its effects are identical to those of a Robin Hood redistribution today; the reason is that a constant path of taxes from tomorrow onwards generates no intertemporal substitution and hence no effects on aggregate variables beyond the effects of a pure redistribution today.

To focus on one source of failure of Ricardian equivalence (sticky prices), we abstracted from another modelling feature that would generate realistic departures from Ricardian equivalence even under flexible prices, namely endogenous investment (for instance, in physical capital). The implications of that assumption have been explored in models with two types of agents elsewhere; see for instance Mankiw (2000). The interaction of endogenous investment and endogenous borrowing limits is certainly worth exploring, but is beyond the scope of this article.

Appendix A. Proof of Proposition 3

Rewrite the system as

$$\begin{bmatrix} \mathbb{E}_t \pi_{t+1} \\ \mathbb{E}_t c_{t+1} \end{bmatrix} = \boldsymbol{\Gamma} \begin{bmatrix} \pi_t \\ c_t \end{bmatrix} + \boldsymbol{\Upsilon}_R \epsilon_t,$$
$$\text{where } \boldsymbol{\Gamma} = \begin{bmatrix} \beta_s^{-1} & -\beta_s^{-1}\kappa \\ \beta_s^{-1}\delta^{-1}(\phi_\pi - 1) & 1 - \beta_s^{-1}\delta^{-1}\kappa(\phi_\pi - 1) \end{bmatrix}, \Upsilon_R = \begin{bmatrix} 0 \\ -\delta^{-1}\eta(1-\rho) \end{bmatrix}. \tag{A.1}$$

The impulse response functions are calculated as:

$$\begin{aligned} \boldsymbol{\Omega} &= [\rho I - \boldsymbol{\Gamma}]^{-1} \boldsymbol{\Upsilon}_R \\ &= \frac{1}{\det} \begin{bmatrix} \rho - 1 + \beta_s^{-1}\kappa\delta^{-1}(\phi_\pi - 1) & -\beta_s^{-1}\kappa \\ \beta_s^{-1}\delta^{-1}(\phi_\pi - 1) & \rho - \beta_s^{-1} \end{bmatrix} \begin{bmatrix} 0 \\ -\delta^{-1}\eta(1-\rho) \end{bmatrix} \\ &= \frac{1}{\det} \begin{bmatrix} \beta_s^{-1}\kappa\delta^{-1}\eta(1-\rho) \\ \delta^{-1}\eta(1-\rho)(\beta_s^{-1} - \rho) \end{bmatrix}. \end{aligned}$$

Appendix B. Proof of Proposition 4

Note that although the exogenous shock has zero persistence, there is endogenous persistence due to the presence of a state variable, public debt; but that endogenous persistence takes a very special form under our assumption that debt is repaid next period: the effects of the shock will live for two periods only. Therefore, to solve the model we must solve for the endogenous variables in periods t and $t+1$. We do this by solving the model backwards as follows: next period's consumption is determined by the Euler equation at $t+1$:

$$c_{t+1} = \mathbb{E}_{t+1}c_{t+2} - \delta^{-1}(i_{t+1} - \mathbb{E}_{t+1}\pi_{t+2}) - \delta^{-1}\eta b_{t+1} + \delta^{-1}\eta(1+\beta_s^{-1})\epsilon_{t+1}^B - \delta^{-1}\eta E_t\epsilon_{t+2}^B,$$

which under zero persistence and the assumption that debt is repaid at $t+1$ (and so $i_{t+1} = \mathbb{E}_{t+1}\pi_{t+2} = \mathbb{E}_{t+1}c_{t+2} = 0$) delivers:

$$c_{t+1} = -\delta^{-1}\eta\beta_s^{-1}\epsilon_t^B = \mathbb{E}_t c_{t+1},$$

where the second equality holds because the shock ϵ_t^B is in the information set at time t. From the Phillips curve at $t+1$, imposing $\mathbb{E}_{t+1}\pi_{t+2} = 0$, we have:

$$\pi_{t+1} = -\kappa\delta^{-1}\eta\beta_s^{-1}\epsilon_t^B = \mathbb{E}_t\pi_{t+1}.$$

The impact multiplier, substituting these expressions in the Euler equation at time t is:

$$c_t = \delta^{-1}\eta[1+\delta^{-1}\kappa(\phi_\pi - 1)\beta_s^{-1}]\epsilon_t^B - \delta^{-1}\eta b_t,$$

and inflation is, from the Phillips curve:

$$\pi_t = \kappa^2\delta^{-2}\eta(\phi_\pi - 1)\beta_s^{-1}\epsilon_t^B - \kappa\delta^{-1}\eta b_t.$$

Appendix C. Solution under a Contemporaneous Taylor Rule

Suppose that the Taylor rule responds to realised inflation:

$$i_t = \phi\pi_t. \tag{C.1}$$

In the case of redistribution with zero persistence, the effects are obtained by merely replacing (C.1) in the IS curve (24):

$$c_t = \frac{\delta^{-1}\eta}{1+\delta^{-1}\phi\kappa}\epsilon_t; \quad \pi_t = \frac{\delta^{-1}\eta\kappa}{1+\delta^{-1}\phi\kappa}\epsilon_t.$$

Redistribution has smaller effects than those obtained under a forward-looking Taylor rule. The reason is that the inflationary effect of redistribution triggers an increase in the real interest rate, which in turn induces savers to consume and work less today (the term $\delta^{-1}\phi\kappa$ in the denominator). This effect disappears when either prices are fixed ($\kappa = 0$ and there is no inflation in equilibrium) or there are no savers ($\lambda \to 1$ implies that $\delta^{-1} \to 0$).

Under public debt with perfect stabilisation ($\phi_B = 1$), the tax process (19) replaced in the IS curve at times t and $t+1$ delivers, respectively:

$$c_t = \mathbb{E}_t c_{t+1} - \delta^{-1}(\phi\pi_t - \mathbb{E}_t\pi_{t+1}) - \delta^{-1}\eta b_t + \delta^{-1}\eta(1+\beta_s^{-1})\epsilon_t^B,$$
$$c_{t+1} = -\delta^{-1}\phi\pi_{t+1} - \delta^{-1}\eta\beta_s^{-1}\epsilon_t^B,$$

while the Phillips curves are

$$\pi_t = \beta\mathbb{E}_t\pi_{t+1} + \kappa c_t,$$
$$\pi_{t+1} = \kappa c_{t+1},$$

where we have accounted for the variables returning to steady state from period $t+2$ onwards. Solving the above system, we obtain:

$$c_{t+1} = -\frac{\delta^{-1}\eta\beta_s^{-1}}{1+\delta^{-1}\phi\kappa}\epsilon_t^B,$$
$$\pi_{t+1} = -\kappa\frac{\delta^{-1}\eta\beta_s^{-1}}{1+\delta^{-1}\phi\kappa}\epsilon_t^B,$$

$$c_t = \frac{\delta^{-1}\eta[1+2\delta^{-1}\phi\kappa+\delta^{-1}\kappa\beta_s^{-1}(\phi-1)]}{(1+\delta^{-1}\phi\kappa)^2}\epsilon_t^B - \frac{\delta^{-1}\eta}{(1+\delta^{-1}\phi\kappa)}b_t,$$
$$\pi_t = \frac{\delta^{-2}\kappa^2\eta[\phi+\beta_s^{-1}(\phi-1)]}{(1+\delta^{-1}\phi\kappa)^2}\epsilon_t^B - \frac{\delta^{-1}\eta\kappa}{(1+\delta^{-1}\phi\kappa)}b_t.$$

The present-value multiplier is:

$$\mathcal{M}_{debt} \equiv \frac{\partial(c_t+\beta_s c_{t+1})}{\partial\epsilon_t^B} = \delta^{-2}\eta\kappa\frac{[\phi+\beta_s^{-1}(\phi-1)]}{(1+\delta^{-1}\phi\kappa)^2}.$$

The effects differ from those obtained under a forward-looking rule (in Proposition 4) as follows. There is still deflation in period $t+1$ for the same reason as under a forward-looking rule (transfer from saver to borrower). As the real interest rate falls with realised deflation, savers react by increasing their consumption at $t+1$ relative to $t+2$ (when the economy returns to steady state). The expected increase in savers' consumption tomorrow implies that an increase in inflation today—coming from the demand effect of redistribution to borrowers in the first period—will trigger a relatively smaller fall in consumption of savers at time t relative to the case of pure redistribution—once again, because of intertemporal substitution. In equilibrium, firms correctly anticipate lower demand in the future and increase prices today by less than they would if redistribution were not 'reversed' in the future; so inflation increases by less, reinforcing the effect described previously. The present-value aggregate consumption multiplier of a debt-financed tax cut is positive in this case too, and has the same interpretation as for a forward-looking rule.

Appendix D. Analytical Solution with Endogenously Persistent Debt $\phi_B < 1$

Replacing the debt accumulation equation (15) into the Euler equation, we obtain:

$$c_t = \mathbb{E}_t c_{t+1} - \delta^{-1}(i_t - \mathbb{E}_t\pi_{t+1}) + \delta^{-1}\eta\phi_B[\beta_s^{-1}(1-\phi_B)-1]b_t + \delta^{-1}\eta(1+\phi_B\beta_s^{-1})\epsilon_t^B. \quad \text{(D.1)}$$

This is a reduced-form IS curve for a given level of public debt; together with (25) and (26) it can be solved to determine consumption and output as a function of outstanding debt and the fiscal shock. The system to be solved is:

$$\begin{bmatrix} \mathbb{E}_t \pi_{t+1} \\ \mathbb{E}_t c_{t+1} \end{bmatrix} = \mathbf{\Gamma} \begin{bmatrix} \pi_t \\ c_t \end{bmatrix} + \mathbf{\Psi} \boldsymbol{b}_t + \mathbf{\Upsilon}_B \boldsymbol{\epsilon}_t^B,$$
$$\text{where } \mathbf{\Gamma} = \begin{bmatrix} \beta_s^{-1} & -\beta_s^{-1}\kappa \\ \beta_s^{-1}\delta^{-1}(\phi_\pi - 1) & 1 - \beta_s^{-1}\delta^{-1}\kappa(\phi_\pi - 1) \end{bmatrix}, \tag{D.2}$$

$$\mathbf{\Psi} = \begin{Bmatrix} 0 \\ \delta^{-1}\eta\phi_B[1 - \beta_s^{-1}(1 - \phi_B)] \end{Bmatrix}; \quad \mathbf{\Upsilon}_B = \begin{bmatrix} 0 \\ -\delta^{-1}\eta(1 + \phi_B^2\beta_s^{-1}) \end{bmatrix} \tag{D.3}$$

Using the method of undetermined coefficients, we can guess and verify that the solution takes the form:

$$\begin{bmatrix} \pi_t \\ c_t \end{bmatrix} = \mathbf{A}_B b_t + \mathbf{A}_\epsilon \epsilon_t^B,$$

which substituted in the original system (using also the public debt dynamics equation) delivers:

$$\mathbf{A}_B(1 - \phi_B)\beta_s^{-1}\boldsymbol{b}_t + \mathbf{A}_B\beta_s^{-1}\boldsymbol{\epsilon}_t^B = \mathbf{\Gamma}\mathbf{A}_B\boldsymbol{b}_t + \mathbf{\Gamma}\mathbf{A}_\epsilon\boldsymbol{\epsilon}_t^B + \mathbf{\Psi}\boldsymbol{b}_t + \mathbf{\Upsilon}_B\boldsymbol{\epsilon}_t^B.$$

Identifying coefficients:

$$\mathbf{A}_B = [(1 - \phi_B)\beta_s^{-1}\mathbf{I} - \mathbf{\Gamma}]^{-1}\mathbf{\Psi},$$
$$\mathbf{A}_\epsilon = \mathbf{\Gamma}^{-1}(\mathbf{A}_B\beta_s^{-1} - \mathbf{\Upsilon}_B).$$

$$\mathbf{A}_B = \frac{1}{\det}\begin{bmatrix} (1 - \phi_B)\beta_s^{-1} - 1 + \beta_s^{-1}\kappa\delta^{-1}(\phi_\pi - 1) & -\beta_s^{-1}\kappa \\ \beta_s^{-1}\delta^{-1}(\phi_\pi - 1) & -\phi_B\beta_s^{-1} \end{bmatrix}\begin{bmatrix} 0 \\ \delta^{-1}\eta\phi_B[1 - \beta_s^{-1}(1 - \phi_B)] \end{bmatrix},$$
$$\mathbf{A}_B = -\frac{\delta^{-1}\eta\phi_B[1 - \beta_s^{-1}(1 - \phi_B)]}{\det}\begin{bmatrix} \beta_s^{-1}\kappa \\ \beta_s^{-1}\phi_B \end{bmatrix},$$
$$\det = (1 - (1 - \phi_B)\beta_s^{-1})\beta_s^{-1}\phi_B + \beta_s^{-2}\kappa\delta^{-1}(\phi_\pi - 1)(1 - \phi_B) > 0$$

The multipliers on consumption are:

$$\frac{\partial c_t}{\partial \epsilon_t^B} = \delta^{-1}\eta\left\{1 + \frac{\phi_B\beta_s^{-1}\delta^{-1}\kappa(\phi_\pi - 1)}{\phi_B[1 - (1 - \phi_B)\beta_s^{-1}] + (1 - \phi_B)\beta_s^{-1}\delta^{-1}\kappa(\phi_\pi - 1)}\right\},$$
$$\frac{\partial c_{t+i}}{\partial \epsilon_t^B} = -\frac{\delta^{-1}\eta\phi_B(1 - (1 - \phi_B)\beta_s^{-1})}{\phi_B[1 - (1 - \phi_B)\beta_s^{-1}] + (1 - \phi_B)\beta_s^{-1}\delta^{-1}\kappa(\phi_\pi - 1)}\phi_B(1 - \phi_B)^{i-1}\beta_s^{-i}, \text{ for } i \geq 1.$$

Appendix E. Dynamic Euler equation tests[23]

This appendix reports results from dynamic Euler equation tests applied to different parameterisations of the model. Details about the procedure can be found in Den Haan (2010).

[23] Prepared with the assistance of Pablo Winant.

Table E1

Baseline Calibration

β_s	0.99
β_b	0.95
χ_s	$(w/c_s)^{\frac{1}{\varphi}}$
χ_b	$(w/c_b)^{\frac{1}{\varphi}}$
ϕ	1
μ	0.2
$\overline{D}$	0.5
$\underline{D}$	0
λ	0.35
φ_i	1.5
κ	58.252
ρ_z	0.979
σ_z	0.0072
A_τ	0.05
ρ_τ	0.9
σ_τ	$A_\tau\sqrt{(1-\rho_\tau^2)}$

E.1. *Baseline calibration*

Table E.1 below displays parameter values under our baseline calibration. A description of the parameters can be found in the main text. Relative to the exercises conducted in the text, we include an additional source of uncertainty: total factor productivity shocks, following an AR(1) process with standard deviation σ_z and persistence ρ_z. In the sticky price version of the model we adopted a specification with quadratic adjustment costs, with parameter κ measuring the degree of price stickiness (the value $\kappa = 58.252$ is consistent with a four quarter rigidity).

E.2. *Sensitivity*

Table E.2 contains a description of the alternative scenarios explored in the simulations.

Table E2

Alternative Calibrations

Name	Difference relative to baseline calibration
baseline	-
low $\overline{D}$	$\overline{D} = 0.05$
high $\overline{D}$	$\overline{D} = 1$
low λ	$\lambda = 0.1$
high λ	$\lambda = 0.45$
high β_b	$\beta_b = 0.98$
higher β_b	$\beta_b = 0.985$
higher β_b + low $\overline{D}$	$\beta_b = 0.985$, $\overline{D} = 0$
higher β_b + high $\overline{D}$	$\beta_b = 0.985$, $\overline{D} = 1$
higher β_b + high λ	$\beta_b = 0.985$, $\lambda = 0.45$
large shock 1	$\rho_\tau = 0$, $\sigma_\tau = 0.01$
large shock 2	$\rho_\tau = 0$, $\sigma_\tau = 0.03$
large shock 3	$\rho_\tau = 0$, $\sigma_\tau = 0.05$

Table E3

Standard Deviations under Baseline Calibration

Flexible price model	
variable	st. dev.
z	0.0072
c,y	0.0072
b	0.0000
r	0.0662
n_s	0.0062
n_b	0.0132
ψ	0.0614
w	0.0072
n	0.0006
c_b	0.0151
c_s	0.0095

Table E4

Standard Deviations under Baseline Calibration

Sticky price model	
variable	st. dev.
z	0.0072
c,y	0.0072
b	0.0000
n_s	0.0075
n_b	0.0133
w	0.0073
ψ	0.0665
n	0.0002
c_b	0.0166
c_s	0.0092

E.3. *Volatilities*

Tables E.3 and E.4 report the implied volatilities (expressed in absolute units) for the growth rates of the endogenous variables under the baseline scenario. Standard deviations are computed from a Monte Carlo experiment over 1000 simulations, with each simulation of horizon 100 periods.

E.4. *Approximation errors*

Approximation errors are computed following the dynamic Euler equation procedure suggested in Den Haan (2010). The procedure can be described as follows.

- For the same draws of the random innovations $(\epsilon_\tau, \epsilon_z)$, two simulations are computed, henceforth referred to as "perturbations" and "exact" respectively.
- In the first simulation, the controls are approximated by a first order Taylor expansion (as in the paper) and the transitions of the states are computed exactly taking these controls as given.
- In the second simulation, the Euler equations are solved exactly to yield the controls used to generate the simulations. In this case, the perturbation

solution is used only indirectly, to approximate future controls while forming the conditional expectations. Numerical integrations using Gauss-Hermite quadratures and 5 nodes along each shock (25 nodes in total) are used.

- Note that at each step the Euler equation of each agent is solved in a non-linear way. In particular, it is not assumed at any point that the constraint is always binding.

Table E5

Dynamic Euler Equation Test. Flexible Price Model

	Average errors in %								
	b	n_s	n_b	ψ	w	n	n_b	n_s	r
baseline	0.0000	0.0003	0.0007	0.0082	0.0000	0.0002	0.0005	0.0001	0.0001
low $\overline{D}$	0.0000	0.0003	0.0007	0.0076	0.0000	0.0002	0.0004	0.0001	0.0001
high $\overline{D}$	0.0000	0.0003	0.0006	0.0089	0.0000	0.0002	0.0005	0.0001	0.0001
low λ	0.0000	0.0001	0.0007	0.0096	0.0000	0.0000	0.0004	0.0000	0.0000
high λ	0.0000	0.0004	0.0006	0.0073	0.0000	0.0003	0.0005	0.0002	0.0002
high β_b	0.0001	0.0003	0.0007	0.0088	0.0000	0.0002	0.0005	0.0001	0.0001
higher β_b	0.0010	0.0003	0.0007	0.0140	0.0000	0.0002	0.0005	0.0001	0.0003
higher β_b + low $\overline{D}$	0.0013	0.0003	0.0007	0.0147	0.0000	0.0002	0.0005	0.0001	0.0002
higher β_b + high $\overline{D}$	0.0000	0.0003	0.0007	0.0075	0.0000	0.0002	0.0004	0.0001	0.0001
higher β_b + high λ	0.0012	0.0005	0.0007	0.0157	0.0000	0.0003	0.0005	0.0002	0.0004
large shock 1	0.0000	0.0000	0.0001	0.0071	0.0000	0.0000	0.0000	0.0000	0.0001
large shock 2	0.0005	0.0002	0.0005	0.0327	0.0000	0.0001	0.0004	0.0001	0.0003
large shock 3	0.0045	0.0010	0.0022	0.1278	0.0000	0.0002	0.0020	0.0009	0.0015

Note: all entries in % except for the real interest rate r in levels.

Table E6

Dynamic Euler Equation Test. Flexible Price Model

	Maximum errors in %								
	b	n_s	n_b	ψ	w	n	c_b	c_s	r
baseline	0.0000	0.0014	0.0037	0.0370	0.0000	0.0012	0.0027	0.0006	0.0005
low $\overline{D}$	0.0000	0.0014	0.0037	0.0347	0.0000	0.0012	0.0026	0.0006	0.0006
high $\overline{D}$	0.0000	0.0014	0.0036	0.0405	0.0000	0.0012	0.0029	0.0005	0.0005
low λ	0.0000	0.0003	0.0038	0.0449	0.0000	0.0003	0.0026	0.0000	0.0001
high λ	0.0000	0.0023	0.0036	0.0321	0.0000	0.0018	0.0028	0.0011	0.0010
high β_b	0.0024	0.0014	0.0039	0.0468	0.0000	0.0012	0.0026	0.0007	0.0010
higher β_b	0.0109	0.0015	0.0042	0.1021	0.0000	0.0012	0.0027	0.0009	0.0021
higher β_b + low $\overline{D}$	0.0141	0.0015	0.0039	0.1040	0.0000	0.0012	0.0031	0.0009	0.0021
higher β_b + high $\overline{D}$	0.0000	0.0014	0.0038	0.0343	0.0000	0.0012	0.0026	0.0007	0.0006
higher β_b + high λ	0.0119	0.0024	0.0038	0.1227	0.0000	0.0017	0.0029	0.0015	0.0032
large shock 1	0.0000	0.0002	0.0005	0.0400	0.0000	0.0001	0.0002	0.0000	0.0003
large shock 2	0.0247	0.0031	0.0068	0.4374	0.0000	0.0006	0.0065	0.0030	0.0057
large shock 3	0.0818	0.0097	0.0207	1.3710	0.0000	0.0014	0.0205	0.0097	0.0184

Note: all entries in % except for the real interest rate r in levels.

Table E7

Frequency of Borrowing Constraint Non-binding. Flexible Price Model

	absolute frequency
baseline	0.0000
low $\overline{D}$	0.0000
high $\overline{D}$	0.0000
low λ	0.0000
high λ	0.0000
high β_b	0.0222
higher β_b	0.1638
higher β_b + low $\overline{D}$	0.1631
higher β_b + high $\overline{D}$	0.0000
higher β_b + high λ	0.1847
large shock 1	0.0000
large shock 2	0.0391
large shock 3	0.1611

For a given sequence of shocks, the comparison of the two simulations is a way to assess how large the approximation errors are and whether or not they accumulate over time. The simulations are performed over a 100-period horizon. For each sequence of shocks the maximum difference between the two simulations is computed. The mean of these maximum error over 1000 draws is reported in the Tables.

It is also informative to check how often the borrowing constraint remains binding in the experiments. We report the percentage of dates over all the simulations for which the constraint remains binding. We find that in the baseline scenario the constraint is always binding, confirming (for reasonably calibrated parameters) the accuracy of the first order approximated solution employed in the main analysis.

Note that to have the constraint binding under flexible prices, we need to assume that:

(*i*) either the discount factors of the two agents are close to each other, or
(*ii*) that the tax (redistributive) shock has a large (and somewhat unrealistic) standard deviation.

In the model with sticky prices, the proportion of dates at which the constraint is binding is higher.

E.4.1. *Flexible prices*

Tables E.5 and E.6 report approximation errors (average and maximum respectively for the flexible price case). Table E.7 reports the frequency with which the borrowing constraint is non-binding under flexible prices.

Table E8

Dynamic Euler Equation Test. Sticky Price Model

	Average errors in %									
	b	n_s	n_b	ψ	w	n	c_b	c_s	i	π
baseline	0.0000	0.0009	0.0008	0.0006	0.0204	0.0005	0.0008	0.0000	0.0000	0.0001
low $\overline{D}$	0.0000	0.0008	0.0008	0.0005	0.0193	0.0005	0.0007	0.0000	0.0000	0.0001
high $\overline{D}$	0.0000	0.0009	0.0008	0.0006	0.0217	0.0005	0.0008	0.0000	0.0000	0.0001
low λ	0.0000	0.0001	0.0007	0.0001	0.0102	0.0001	0.0005	0.0000	0.0000	0.0000
high λ	0.0000	0.0028	0.0008	0.0022	0.0616	0.0015	0.0025	0.0001	0.0000	0.0002
high β_b	0.0001	0.0009	0.0008	0.0006	0.0196	0.0005	0.0008	0.0000	0.0000	0.0001
higher β_b	0.0005	0.0009	0.0008	0.0006	0.0234	0.0005	0.0007	0.0001	0.0000	0.0001
higher β_b + low $\overline{D}$	0.0005	0.0009	0.0008	0.0006	0.0245	0.0005	0.0008	0.0001	0.0000	0.0001
higher β_b + high $\overline{D}$	0.0000	0.0009	0.0007	0.0006	0.0197	0.0005	0.0008	0.0001	0.0000	0.0001
higher β_b + high λ	0.0006	0.0027	0.0008	0.0021	0.0541	0.0014	0.0023	0.0001	0.0001	0.0003
large shock 1	0.0000	0.0004	0.0002	0.0003	0.0241	0.0002	0.0003	0.0000	0.0000	0.0000
large shock 2	0.0045	0.0045	0.0027	0.0043	0.2913	0.0021	0.0065	0.0001	0.0003	0.0003
large shock 3	0.0147	0.0115	0.0071	0.0114	0.7574	0.0053	0.0174	0.0004	0.0011	0.0008

Note: all entries in % except for the nominal interest rate i in levels.

Table E9

Dynamic Euler Equation Test. Sticky Price Model

	Maximum errors in %									
	b	n_s	n_b	ψ	w	n	c_b	c_s	i	π
baseline	0.0000	0.0047	0.0043	0.0029	0.1093	0.0031	0.0047	0.0002	0.0000	0.0003
low $\overline{D}$	0.0000	0.0044	0.0045	0.0026	0.1005	0.0031	0.0045	0.0002	0.0000	0.0002
high $\overline{D}$	0.0000	0.0048	0.0043	0.0030	0.1163	0.0032	0.0049	0.0002	0.0000	0.0003
low λ	0.0000	0.0006	0.0040	0.0004	0.0479	0.0005	0.0026	0.0000	0.0000	0.0000
high λ	0.0000	0.0162	0.0046	0.0127	0.3626	0.0092	0.0151	0.0003	0.0000	0.0011
high β_b	0.0021	0.0045	0.0044	0.0029	0.1091	0.0030	0.0045	0.0001	0.0002	0.0003
higher β_b	0.0059	0.0043	0.0047	0.0029	0.1403	0.0028	0.0044	0.0002	0.0004	0.0004
higher β_b + low $\overline{D}$	0.0053	0.0044	0.0047	0.0031	0.1431	0.0028	0.0045	0.0002	0.0004	0.0004
higher β_b + high $\overline{D}$	0.0000	0.0046	0.0043	0.0028	0.1056	0.0032	0.0046	0.0002	0.0000	0.0003
higher β_b + high λ	0.0061	0.0149	0.0049	0.0114	0.2914	0.0086	0.0133	0.0004	0.0013	0.0016
large shock 1	0.0026	0.0030	0.0017	0.0023	0.1915	0.0016	0.0031	0.0001	0.0002	0.0002
large shock 2	0.0543	0.0311	0.0186	0.0317	2.2804	0.0139	0.0482	0.0013	0.0040	0.0026
large shock 3	0.1174	0.0604	0.0381	0.0616	4.5187	0.0267	0.0951	0.0029	0.0086	0.0055

Note: all entries in % except for the nominal interest rate i in levels.

E.4.2. *Sticky prices*

Tables E.8 and E.9 report approximation errors (average and maximum respectively for the sticky price case. Table E.10 reports the frequency with which the borrowing constraint is non-binding under sticky prices.

Finally, Figures E.1 to E.3 report the Lagrange multiplier and debt under the two simulations, for three scenarios respectively: baseline, higher β_b, and large shock 3.

Table E10

Frequency of Borrowing Constraint Non-binding. Sticky Price Model

	absolute frequency
baseline	0.0000
low $\overline{D}$	0.0000
high $\overline{D}$	0.0000
low λ	0.0000
high λ	0.0003
high β_b	0.0423
higher β_b	0.1832
higher β_b + low $\overline{D}$	0.1772
higher β_b + high $\overline{D}$	0.0000
higher β_b + high λ	0.2497
large shock 1	0.0091
large shock 2	0.2551
large shock 3	0.4048

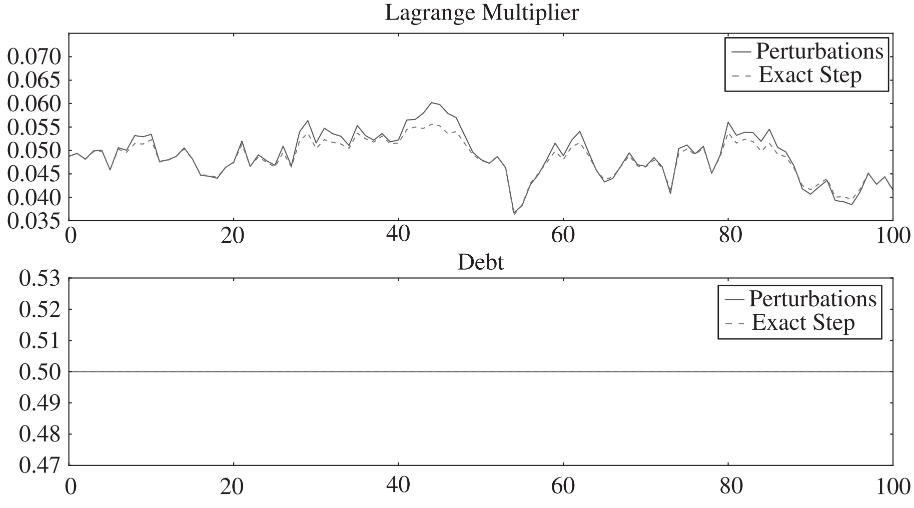

Fig. E1. *Lagrange Multiplier and Private Debt under the Two Simulations, Baseline Calibration*

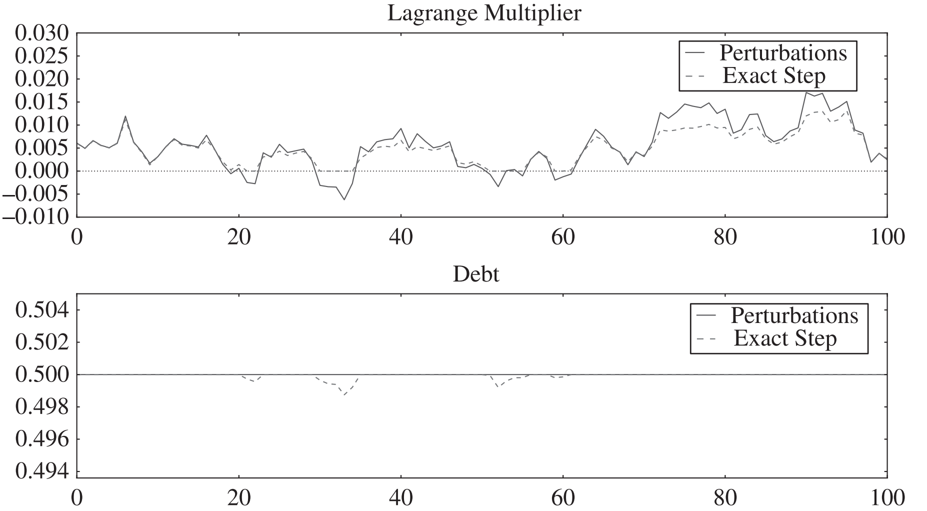

Fig. E2. *Lagrange Multiplier and Private Debt under the Two Simulations, "Higher β_b" Calibration*

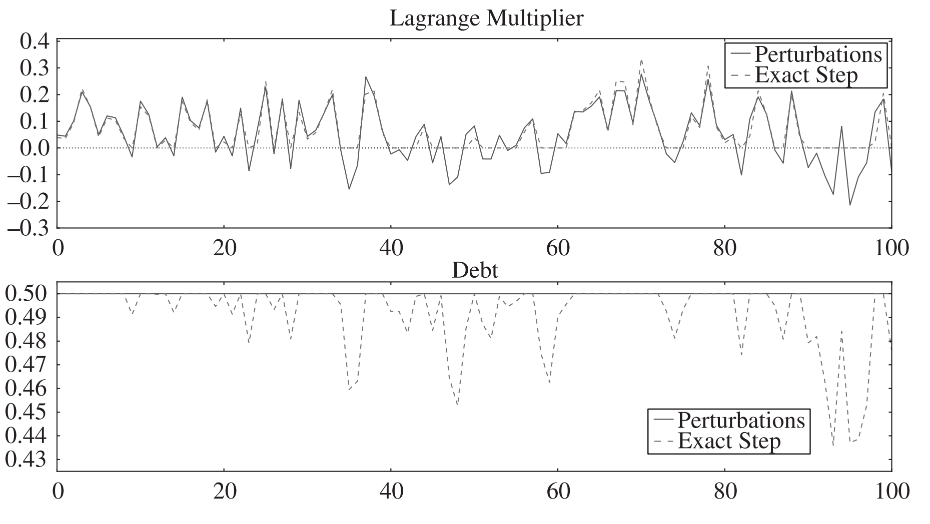

Fig. E3. *Lagrange Multiplier and Private Debt under the Two Simulations, "big shock 3" Calibration*

Appendix F. Robustness

This appendix reports the effects on the responses of aggregate consumption, inflation and real government debt to a uniform tax cut of varying alternative parameter values. In all cases the responses are derived conditional on the log-linearised solution of the model. This analysis shows that parameters such as the share of borrowers in the population, λ, and ϕ, the inverse elasticity of labour supply, can have a sizeable effect on the aggregate dynamics; whereas other parameters, such as the steady state ratio of public and private debt to output have only a negligible effect.

In all cases the baseline parameterisation is log-consumption utility, $\phi = 1$, $\theta = 0.75$, $\varepsilon = 6$, $\beta_s = 0.99, \beta_b = 0.95, \phi_\pi = 1.5$, $\lambda = 0.35$, $\phi_B = 0.1$ (see main text for a meaning of the parameters). Unless reported otherwise, the steady state share of government

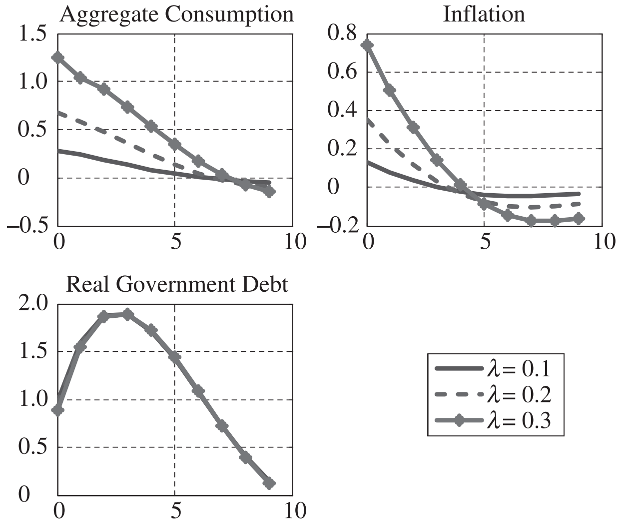

Fig. F1. *Effect of Varying the Share of Borrowers λ on the Responses of Aggregate Consumption, Inflation and Real Government Debt to a Uniform Tax Cut*

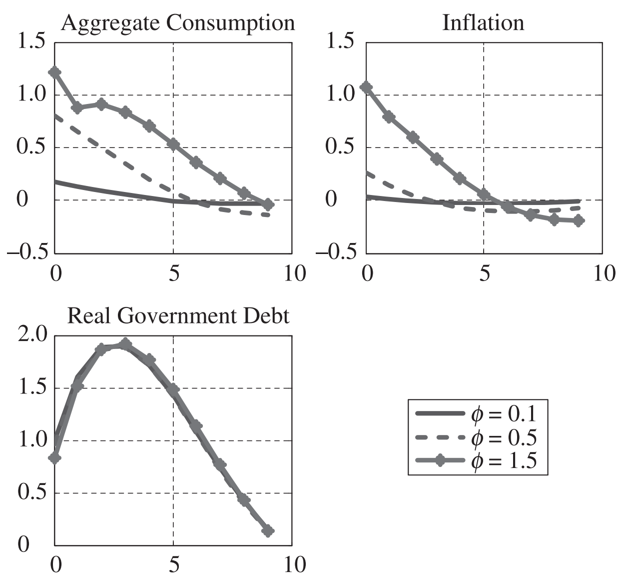

Fig. F2. *Effect of Varying the Inverse Elasticity of Labor Supply ϕ on the Responses of Aggregate Consumption, Inflation and Real Government Debt to a Uniform Tax Cut*

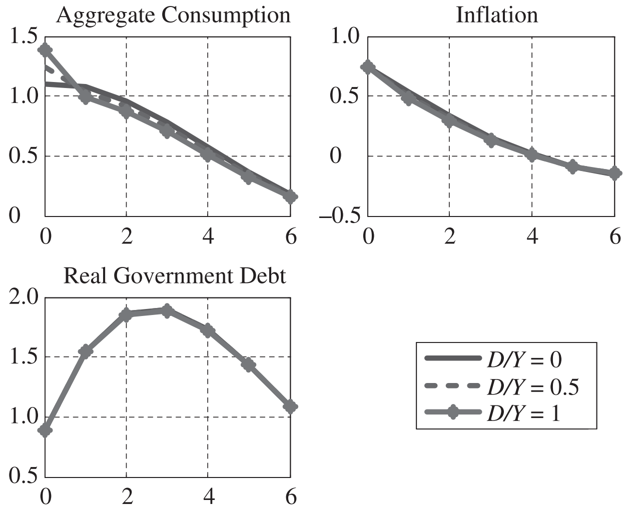

Fig. F3. *Effect of Varying the Steady State Ratio of Private Debt to Output, D/Y, on the Responses of Aggregate Consumption, Inflation and Real Government Debt to a Uniform Tax Cut*

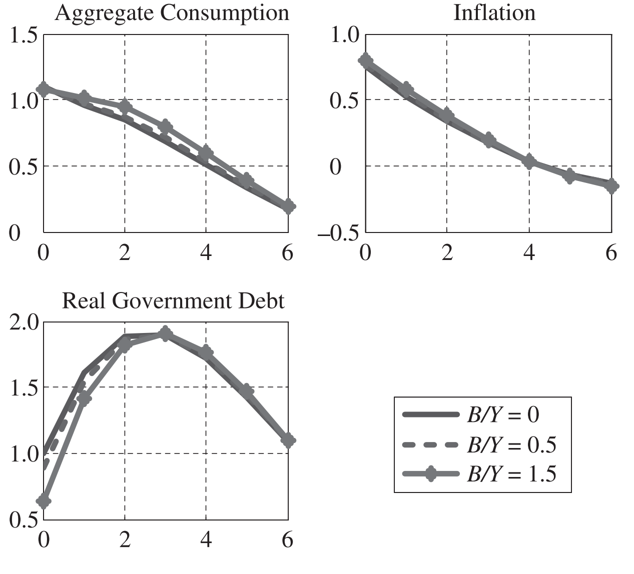

Fig. F4. *Effect of Varying the Steady State Ratio of Public Debt to Output, B/Y, on the Responses of Aggregate Consumption, Inflation and Real Government Debt to a Uniform Tax Cut*

spending in output is 0.2, the steady state ratio of government debt to output is 0.5, and the steady state ratio of private debt to output is 0.3. On the horizontal axis periods are measured in quarters. Monetary policy follows the Taylor-type log-linear rule $i_t = \phi_\pi \mathbb{E}_t(\pi_{t+1})$. Tax policy follows the rule $t_t = \phi_B b_t - \epsilon_t$ (see text for more details).

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